Примеры решения задач

1. $$\int_0^{\frac{\pi}{2}} \cos x\,dx = \sin x \Big|_0^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin 0 = 1.$$

2. $$\int_2^4 \left(x^3 + x\right)dx = \left(\frac{x^4}{4} + \frac{x^2}{2}\right)\Bigg|_2^4 = 64 + 8 - \left(4 + 2\right) = 66.$$

3. $$\int_1^e \frac{dx}{2x-1} = \frac{1}{2}\ln\left|2x-1\right|\Big|_1^e = \frac{1}{2}\ln\left(2e-1\right) - \ln 1 = \frac{1}{2}\ln\left(2e-1\right).$$

4. $$\int_0^{\frac{\pi}{2}} \cos^2 x \sin x\,dx = -\int_0^{\frac{\pi}{2}} \cos^2 x d\left(\cos x\right) = -\frac{1}{3}\cos^3 x\Big|_0^{\frac{\pi}{2}} = -\frac{1}{3}\left(\cos^3\frac{\pi}{2} - \cos^3 0\right) = \frac{1}{3}.$$

5. $$\int_0^a \sqrt{a^2 - x^2}\,dx = \left[\begin{array}{l} x = a\sin t, \Rightarrow t = \arcsin\frac{x}{a}, \Rightarrow dx = a\cos t dt, \\ \alpha = \arcsin 0 = 0, \beta = \arcsin 1 = \frac{\pi}{2} \end{array}\right] =$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2\sin^2 t} \cdot a\cos t dt = a^2\int_0^{\frac{\pi}{2}} \sqrt{1 - \sin^2 t} \cdot \cos t dt = a^2\int_0^{\frac{\pi}{2}} \cos^2 t dt = \frac{a^2}{2}\int_0^{\frac{\pi}{2}} \left(1 + \cos 2t\right)dt =$$

$$= \frac{a^2}{2}\left(t + \frac{1}{2}\sin 2t\right)\Bigg|_0^{\frac{\pi}{2}} = \frac{a^2}{2}\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi - \frac{1}{2}\sin 0\right) = \frac{a^2\pi}{4}.$$

6. $$\int_0^5 x\sqrt{x+4}\,dx = \left[\begin{array}{l} x + 4 = t^2, \Rightarrow x = t^2 - 4, \Rightarrow dx = 2tdt, \\ t = \sqrt{x+4}, \alpha = \sqrt{4} = 2, \beta = \sqrt{9} = 3 \end{array}\right] = \int_2^3 \left(t^2 - 4\right) \cdot t \cdot 2tdt =$$

$$= 2\int_2^3 \left(t^4 - 4t^2\right)dt = 2\left(\frac{t^5}{5} - \frac{4t^3}{3}\right)\Bigg|_2^3 = 2\left(\frac{243}{5} - \frac{108}{3} - \frac{32}{5} + \frac{32}{3}\right) = 33\frac{11}{15}.$$

7. $$\int_1^e x\ln x\,dx = \left[\begin{array}{l|l} u = \ln x & du = \frac{dx}{x} \\ dv = xdx & v = \frac{x^2}{2} \end{array}\right] = \frac{1}{2}x^2\ln x\Big|_1^e - \frac{1}{2}\int_1^e x\,dx = \frac{1}{2}\left(e^2\ln e - \ln 1\right) - \frac{1}{4}x^2\Big|_1^e =$$

$$= \frac{1}{2}e^2 - \frac{1}{4}\left(e^2 - 1\right) = \frac{1}{4}\left(e^2 + 1\right).$$